Computing the Shielding Effectiveness of Thin Screens by the Finite Element Method

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***Abstract*— In order to protect electronic devices and dedicated areas against exposure of RF-electromagnetic radiation very thin screening materials will be employed. Single and multi-layered metallic foils as well as metamaterials will be utilized in favor. The ambition of this contribution lies in the computation of such thin screens with the finite element (FE) method. To overcome numerical difficulties due to the very thin layers, analytical descriptions of the layers will be implemented in the FE-formulation. An improvement of the mesh truncation will be achieved by prescribing improved impedance boundary conditions containing a surface operator. A given antenna and screen arrangement has been measured and a comparison to the results obtained numerically will demonstrate the efficiency and reliability of the way suggested.**

Introduction

In general, highly sensitive electronic equipment will be protected against electromagnetic radiation caused inducement with the aid of cases coated by thin metallic foils. Computing such configurations with the FE-method is attended by the disadvantage of large jumps in the element size. Hence numerically bad conditioned sets of equations to be solved will occur. Especially in cases where large distances have to be considered, a huge effort in a proper discretization is enforced. To overcome this, it is the aim to avoid modeling the thin layers with FEs. An analytical description of the thin layers instead can be used as shown in [1]. In Fig. 1 the general arrangement is illustrated.

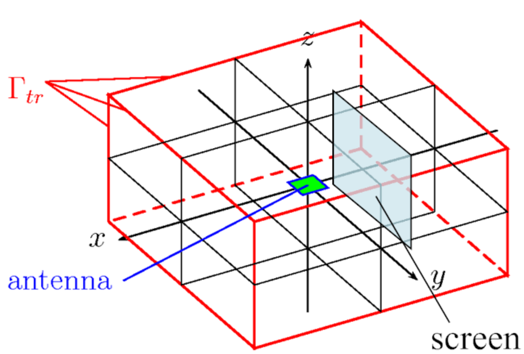

Fig. 1. Basic arrangement, antenna in point of origin, domain truncated by boundary Γ_{tr}.

An antenna is situated in the origin of the co-ordinate system. At a distinct distance away the screen is placed. It may consist of several layers and of variant materials. This arrangement is surrounded by air and the FE-mesh is truncated by the surface labeled with Γ_{tr}. In this contribution a special focus has been cast in providing appropriate boundary conditions for that.

The variant kinds of mesh truncation when using finite elements always represent a compromise between sufficient accuracy and smallest possible computation effort. The use of absorbing boundaries (ABC) has been improved seriously by some iterative procedures, as reported eg. in [2], [3] and [4]. Multiple calculations of the same problem boost the effort and extend the computation time, thereby. The application of perfectly matched layers (PMLs), as suggested e.g. in [5], suffers from a condition number degraded set of equations system to be solved. Hence the convergence while applying an iterative solving procedure becomes worse.

Along the truncation boundaries (Γ_{tr}) the propagation direction can be determined by the location of the radiating antenna and the truncation surface Γ_{tr}, only. The knowledge of the incident angle of the outgoing wave all-over Γ_{tr} enables the design of more precise conditions between the tangential field vectors $\vec{E}_t$ and $\vec{H}_t$ on it.

Surface operator containing boundary conditions

In finding improved boundary conditions for the truncation surface Γ_{tr} we begin with the Maxwell equations

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}, \qquad \nabla \times \vec{H} = j\omega\epsilon\vec{E}. \tag{1}$$

Therein, the field vectors $\vec{E}$ and $\vec{H}$ as well as the ∇ operator can be split into a normal and two orthogonal tangential components. The normal component will be subscribed by n and the tangential vector by t. It follows:

$$\vec{E} = \vec{E}_t + \vec{n}\,E_n,\ \vec{H} = \vec{H}_t + \vec{n}\,H_n,\ \nabla = \nabla_t + \frac{\partial}{\partial n}\vec{n}. \tag{2}$$

With this relations the normal components of the fields, E_n and H_n, can be eliminated in (1) to attain

$$\frac{\partial(\vec{n}\times\vec{E}_t)}{\partial n} = -j\omega\mu\vec{H}_t - \frac{1}{j\omega\epsilon}\nabla_t \times (\nabla_t \times \vec{H}_t) \tag{3}$$

$$\frac{\partial(\vec{n}\times\vec{H}_t)}{\partial n} = j\omega\epsilon\vec{E}_t + \frac{1}{j\omega\mu}\nabla_t \times (\nabla_t \times \vec{E}_t). \tag{4}$$

These equations are commonly valid, consequently on the truncation surfaces, too. On Γ_{tr} the outgoing wave can

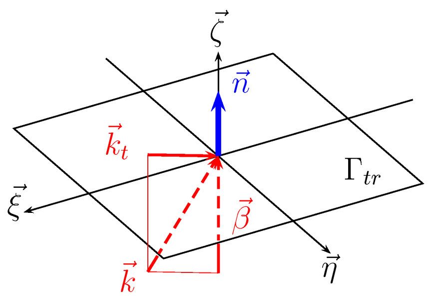

Fig. 2. Outgoing wave at a point on Γ_{tr}.

be represented by the wave vector $\vec{k}$. Due to the knowledge of the incident angle on Γ_{tr}, $\vec{k}$ can be decomposed into a tangential vector $\vec{k_t}$ and the perpendicular one $\vec{\beta}$, corresponding to

$$\vec{k} = \vec{k_t} + \vec{\beta}, \qquad \beta = \pm\sqrt{k^2 - k_t^2}, \qquad k = \omega\sqrt{\mu\epsilon}. \quad (5)$$

To get rid of the derivative $\frac{\partial}{\partial n}$ on the left hand side in (4), the integration along ζ (n-direction) over the positive half space must be performed. Assuming a physical and lossy media the field decays to zero at infinity, which results into:

$$\int_{\zeta=0}^{\infty} \vec{H}_{t_0} e^{-j\beta\zeta} d\zeta = \frac{1}{j\beta}\vec{H}_{t_0}, \int_{\zeta=0}^{\infty} \vec{E}_{t_0} e^{-j\beta\zeta} d\zeta = \frac{1}{j\beta}\vec{E}_{t_0}. \quad (6)$$

$\vec{H}_{t_0}$ and $\vec{E}_{t_0}$ are the tangential field vectors at $\zeta = 0$. Together with (5) the relation (4) changes to

$$\vec{n} \times \vec{H}_{t_0} = \frac{\omega\epsilon\vec{E}_{t_0}}{\sqrt{k^2 - k_t^2}} - \frac{\nabla_t \times (\nabla_t \times \vec{E}_{t_0})}{\omega\mu\sqrt{k^2 - k_t^2}}. \quad (7)$$

Transverse components of the outgoing wave may be transformed into the Fourier domain ($e^{-j\vec{k}_t\cdot\vec{r}}$ -terms), only to see, that its tangential derivatives can be expressed as $\nabla_t = -j\vec{k_t}$. The substitution in (7) leads to

$$\vec{n} \times \vec{H}_{t_0} = \frac{\omega\epsilon\vec{E}_{t_0}}{\sqrt{k^2 - k_t^2}} + \frac{\vec{k_t} \times (\vec{k_t} \times \vec{E}_{t_0})}{\omega\mu\sqrt{k^2 - k_t^2}}. \quad (8)$$

This relation between the tangentials of $\vec{E}_{t_0}$ and $\vec{H}_{t_0}$ now can be used to model the so called surface operator boundary conditions (SOBC) on Γ_{tr}.

Implementation in a FEM-formulation

When applying the Galerkin method to the well known $\vec{A}, v$-formulations [6], the $\vec{n} \times \vec{H}_t$ term occurs, hence (8) can be implemented directly.

The weak form of the vectorial equation follows to

$$-\int_{\Omega} \nabla \times \vec{N_i} \cdot \frac{1}{\mu}\nabla \times \vec{A} d\Omega + \int_{\Gamma_H} \vec{N_i} \cdot \underbrace{(\vec{n} \times (\frac{1}{\mu}\nabla \times \vec{A}))}_{\vec{n}\times\vec{H}} d\Gamma$$
$$+ \int_{\Omega} \vec{N_i} \cdot (\sigma + j\omega\epsilon) j\omega(\vec{A} + \nabla v) d\Omega = 0. \quad (9)$$

On the Neumann boundary Γ_H the presented conditions for the mesh truncation can be prescribed. So, the underbraced term directly may be substituted by (8). Thereby, the Neumann boundary Γ_H corresponds to the truncation surface Γ_{tr}. Expressing the field $\vec{E}_t$ by the potentials, according to

$$\vec{E}_t = -j\omega(\vec{A}_t + \nabla_t v) \quad (10)$$

and setting the electric scalar v to zero ends up in a symmetric matrix to be solved.

Conclusion

At first the disadvantage of modeling very thin layers with finite elements has been overcome by applying an analytical network equations representation of the layers. Hence from that point of view the condition number of the resulting equations system will not suffer any longer. An improved mesh truncation condition has been proposed. Advantage has been taken from the fact, that along the truncation boundaries the incident angle of the outgoing wave is known. Thus, improved absorbing conditions, containing surface operators have been implemented in an integral term over the truncation boundary. Compared to the PML mesh truncation a remarkably smaller domain has to be computed and again, a well better conditioned equations system will be obtained. Comparison to measurements will be given in the full paper version.

References

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